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**Section I True / False questions (5 points each)**

1. \_\_\_\_\_ Only universal and existential WFFs have instances.
2. \_\_\_\_\_ A WFF in predicate logic may contain a free variable.
3. \_\_\_\_\_ If a WFF begins with the symbols “ $\forall x$ ”, then it must be an existential.
4. \_\_\_\_\_ All valid arguments have a countermodel.

**Section II Mark the correct completion (5 points each)**

1. The condition on  $\forall I$  requires that ...
  - (a) \_\_\_\_\_ the instantial name must occur in at least one of the sentences in the assumption of the line to which one applies the rule.
  - (b) \_\_\_\_\_ there is no condition on the application of  $\forall I$ .
  - (c) \_\_\_\_\_ the instantial name cannot occur in any sentence in the assumption set of the line to which one applies the rule.
  - (d) \_\_\_\_\_ a free variable must be used in place of an instantial name.
  - (e) \_\_\_\_\_ the instantial name be used in the sentence which results from the application of the rule.
  
2. The sentence  $\forall x(Fx \rightarrow \sim(\exists yGy \ \& \ R))$  is a ...
  - (a) \_\_\_\_\_ existential
  - (b) \_\_\_\_\_ conditional
  - (c) \_\_\_\_\_ negation
  - (d) \_\_\_\_\_ universal
  - (e) \_\_\_\_\_ conjunction

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3. The following is NOT a condition on the application of  $\exists E$  ...
- (a) \_\_\_\_\_ the instantial name cannot occur in the line that motivates the assumption to be discharged.
  - (b) \_\_\_\_\_ the instantial name cannot occur in the line containing the sentence which is repeated.
  - (c) \_\_\_\_\_ the instantial name must occur in the line which is repeated.
  - (d) \_\_\_\_\_ the instantial name cannot occur in the assumption set of the line containing the sentence which is repeated save for the assumption itself.
4. A finite interpretation may contain all but ...
- (a) \_\_\_\_\_ a universe
  - (b) \_\_\_\_\_ predicate extensions
  - (c) \_\_\_\_\_ truth value specifications
  - (d) \_\_\_\_\_ a proof

**Section III Translations (5 points each)**

**Using the following translation scheme, construct a strictly correct translations that includes all parentheses.**

$Bx =$  'x is a book'

$Hx =$  'x is a hardback'

$Px =$  'x is a paperback'

$Ex =$  'x exists'

$Lxy =$  'x is longer than y'

$a =$  Logic Primer

$b =$  'Crime and Punishment'

- 1) Among books, only paperback and hardback exist.

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2) All books are paperbacks.

3) Crime and Punishment is longer than the Logic Primer, only if Crime and Punishment is a hardback.

4) Not all books are hardback if paperbacks exist.

**Section IV Proofs (8 points each)**

**Give a proof for each of the following sequents. You may use both primitive and derived rules.**

1.  $\forall x(Fx \vee Gx), \forall x(Gx \rightarrow Hx), \exists x \sim Fx \vdash \exists x Hx$

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2.  $\forall x(Px \rightarrow (Qx \ \& \ Rx)), \exists xPx \rightarrow \forall x\sim Rx \vdash \sim\exists xPx$

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**Section V Finite Interpretations (2 points each)**

**For each of the sentences below, indicate whether it is true or false in this finite interpretation:**

U: {a, b, c}

F: {a}

G: {a, b, c}

H: {<a,b>, <b,b>}

1. \_\_\_\_\_ ( $Hba \rightarrow \sim Gb$ )
2. \_\_\_\_\_ ( $\exists x(Fx \& \sim Gx)$ )
3. \_\_\_\_\_ ( $\forall xGx \rightarrow \forall xFx$ )
4. \_\_\_\_\_ ( $\sim \exists xHxx$ )

**Section VI Finite Countermodels (6 points)**

**Construct a counter-model for the following sequent. Be sure to show your work.**

$\exists x(Px \& Rx), \exists x(Sx \& Rx) \vdash \exists x(Px \& Sx)$